

# Mathematics assessed student work

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# Diploma Programme

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## IB mission statement

The International Baccalaureate aims to develop inquiring, knowledgeable and caring young people who help to create a better and more peaceful world through intercultural understanding and respect.

To this end the organization works with schools, governments and international organizations to develop challenging programmes of international education and rigorous assessment.

These programmes encourage students across the world to become active, compassionate and lifelong learners who understand that other people, with their differences, can also be right.



# IB learner profile

**The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.**

**As IB learners we strive to be:**

## **INQUIRERS**

We nurture our curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life.

## **KNOWLEDGEABLE**

We develop and use conceptual understanding, exploring knowledge across a range of disciplines. We engage with issues and ideas that have local and global significance.

## **THINKERS**

We use critical and creative thinking skills to analyse and take responsible action on complex problems. We exercise initiative in making reasoned, ethical decisions.

## **COMMUNICATORS**

We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups.

## **PRINCIPLED**

We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.

## **OPEN-MINDED**

We critically appreciate our own cultures and personal histories, as well as the values and traditions of others. We seek and evaluate a range of points of view, and we are willing to grow from the experience.

## **CARING**

We show empathy, compassion and respect. We have a commitment to service, and we act to make a positive difference in the lives of others and in the world around us.

## **RISK-TAKERS**

We approach uncertainty with forethought and determination; we work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

## **BALANCED**

We understand the importance of balancing different aspects of our lives—intellectual, physical, and emotional—to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live.

## **REFLECTIVE**

We thoughtfully consider the world and our own ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development.

**The IB learner profile represents 10 attributes valued by IB World Schools. We believe these attributes, and others like them, can help individuals and groups become responsible members of local, national and global communities.**

## Overview

The intention of this section is to demonstrate the overall marking standards that are required, as well as to illustrate how the achievement levels for each criterion should be awarded. The assessment was undertaken by experienced moderators, some of whom have been involved in the development of the new internal assessment criteria.

The comments and marks provided for criteria A–E are those of these moderators. All the explorations have been marked using both the standard level (SL) and higher level (HL) criteria. The requirements for criteria A–D are identical for SL and HL; it is only criterion E (use of mathematics) that is different. It is essential that teachers of both subjects and levels standardize their marking of the exploration before submitting their marks.

Some of the explorations have separate comments for mathematics: analysis and approaches and for mathematics: applications and interpretation. This is to illustrate how the marking may be affected based on whether a student has met the mathematics used during their course or has chosen to use mathematics not from their course. Generally, this may affect the mark given in criterion C (personal engagement).

Teachers may simply wish to see how an exploration was marked. Using the links, teachers can navigate to the comments and to the annotated student work. The annotated work shows, for each criterion, where in the exploration evidence can be seen for reaching a decision on awarding the mark for that criterion.

Alternatively, teachers may wish to mark the student work themselves. Using the links, the student work can also be viewed in its original format. Teachers can then compare their own marking to that of the moderator and go on to look at the annotated student work.

The comments should be read in conjunction with the explorations and the annotations on the explorations.

The table below categorizes the explorations by the “best fit” for the topic of the exploration and shows the marks given for SL (first number) and HL (second number). When a cell is split in two, the top part shows the marks for mathematics: analysis and approaches (analysis) and the bottom part shows the marks for mathematics: applications and interpretation (applications).

Example number	Title	Number and algebra	Functions (and modelling)	Geometry and trigonometry	Probability and statistics	Calculus	Other
1	Horse's jump		SL 11 marks HL 9 marks				
2	Infant mortality				SL 16 marks HL 14 marks		
3	Packaging and geometrical shapes			SL 14 marks HL 12 marks			
4	Rowing speeds				SL 16 marks HL 15 marks		
5	Social media				SL 9 marks HL 7 marks		
6	Path of quickest descent		SL 9 marks				

Example number	Title	Number and algebra	Functions (and modelling)	Geometry and trigonometry	Probability and statistics	Calculus	Other
			HL 8 marks				
7	Bayes' Theorem and baseball				SL 10 marks HL 9 marks		
8	Modelling stools					SL 13 marks HL 11 marks	
9	The Cantor set						SL 16 marks HL 15 marks
10	Human population growth		Analysis SL16 marks Analysis HL 15 marks Applications SL16 marks Applications HL 15 marks				
11	Pursuit curves					SL 15 marks HL 14 marks	
12	Microwave popcorn		SL 20 marks HL 20 marks				
13	Ice cream					SL 15 marks HL 13 marks	
14	Zeno's arrow paradox					SL 18 marks HL 17 marks	
15	Chinese remainder theorem	SL 12 marks HL 11 marks					
16	Solar panels			SL 19 marks HL 19 marks			
17	Hyperboloids			SL 19 marks HL 19 marks			
18	Prime number theories	SL 10 marks HL 8 marks					
19	Optimisation at the cottage					SL 11 marks HL 9 marks	
20	Encrypting messages	Analysis SL16 marks Analysis HL 14 marks					

Example number	Title	Number and algebra	Functions (and modelling)	Geometry and trigonometry	Probability and statistics	Calculus	Other
		Applications SL 17 marks Applications HL 15 marks					
21	Gabriel Dawe's rainbow coloured curves		SL 19 marks HL 18 marks				
22	Titus tunnel bridge		SL 19 marks HL 18 marks				
23	Transport fare charges in Madrid		SL 15 marks HL 14 marks				
24	Golf and kinematics		SL 7 marks HL 7 marks				
25	Mathematics in barcodes	SL 13 marks HL 12 marks					
26	Probability in the courtroom				SL 19 marks HL 18 marks		
27	Converting piano music to guitar music		SL 11 marks HL 10 marks				
28	The Chinese postman problem: analysis	Analysis SL 14 marks HL 14 marks					
29	Real-world applications of the prisoner's dilemma				SL 10 marks HL 9 marks		



## Example 1: Horse's jump

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 2: Infant mortality

**Subject:** Mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

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## Example 3: Packaging and geometrical shapes

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 4: Rowing speeds

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level and (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

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## Example 5: Social media

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

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## Example 6: Path of quickest descent

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 7: Bayes' Theorem and baseball

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

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## Example 8: Modelling stools

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)



## Example 9: The Cantor set

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 10: Human population growth

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 11: Pursuit curves

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 12: Microwave popcorn

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 13: Ice cream

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 14: Zeno's arrow paradox

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 15: Chinese remainder theorem

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 16: Solar panels

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)



## Example 17: Hyperboloids

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 18: Prime number theories

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

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## Example 19: Optimisation at the cottage

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment (IA), standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 20: Encrypting messages

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

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## Example 21: Gabriel Dawe's rainbow coloured curves

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 22: Titus tunnel bridge

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 23: Transport fare charges in Madrid

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 24: Golf and kinematics

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)



## Example 25: Mathematics in barcodes

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 26: Probability in the courtroom

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 27: Converting piano music to guitar music

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 28: The Chinese postman problem: analysis

**Subject:** Mathematics: analysis and approaches

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

[Student work \(PDF\)](#)

[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)

## Example 29: Real-world applications of the prisoner's dilemma

**Subject:** Mathematics: analysis and approaches and mathematics: applications and interpretation

**Paper component:** Internal assessment, standard level (SL) and higher level (HL)

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[Annotated student work \(PDF\)](#)

[Examiner's comments coversheet \(PDF\)](#)